Pearson Edexcel

Mark Scheme (Results)

Summer 2023

Pearson Edexcel International GCSE
In Further Pure Mathematics (4PM1)
Paper 02R

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the last candidate in exactly the same way as they mark the first.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme - not according to their perception of where the grade boundaries may lie.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification/indicative content will not be exhaustive.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, a senior examiner must be consulted before a mark is given.
- Crossed out work should be marked **unless** the candidate has replaced it with an alternative response.

- **Types of mark**
    - o M marks: method marks
    - o A marks: accuracy marks
    - o B marks: unconditional accuracy marks (independent of M marks)

- **Abbreviations**
    - o cao – correct answer only
    - o ft – follow through
    - o isw – ignore subsequent working
    - o SC - special case
    - o oe – or equivalent (and appropriate)
    - o dep – dependent
    - o indep – independent
    - o awrt – answer which rounds to
    - o eeoo – each error or omission

- **No working**

If no working is shown then correct answers normally score full marks
If no working is shown then incorrect (even though nearly correct) answers score no marks.

- **With working**
If the final answer is wrong, always check the working in the body of the script (and on any diagrams), and award any marks appropriate from the mark scheme.
If it is clear from the working that the “correct” answer has been obtained from incorrect working, award 0 marks.
If a candidate misreads a number from the question. Eg. Uses 252 instead of 255; method marks may be awarded provided the question has not been simplified. Examiners should send any instance of a suspected misread to review.
If there is a choice of methods shown, then award the lowest mark, unless the answer on the answer line makes clear the method that has been used.
If there is no answer achieved then check the working for any marks appropriate from the mark scheme.

- **Ignoring subsequent work**
It is appropriate to ignore subsequent work when the additional work does not change the answer in a way that is inappropriate for the question: eg. Incorrect cancelling of a fraction that would otherwise be correct.
It is not appropriate to ignore subsequent work when the additional work essentially makes the answer incorrect eg algebra.
Transcription errors occur when candidates present a correct answer in working, and write it incorrectly on the answer line; mark the correct answer.

- **Parts of questions**
Unless allowed by the mark scheme, the marks allocated to one part of the question CANNOT be awarded to another.

General Principles for Further Pure Mathematics Marking

(but note that specific mark schemes may sometimes override these general principles)

Method mark for solving a 3 term quadratic equation:

1. Factorisation:

$(x^2+bx+c)=(x+p)(x+q)$, where $|pq|=|c|$ leading to $x=\ldots.$

$(ax^2+bx+c)=(mx+p)(nx+q)$ where $|pq|=|c|$ and $|mn|=|a|$ leading to $x=\ldots.$

2. Formula:

Attempt to use the **correct** formula (shown explicitly or implied by working) with values for *a*, *b* and *c*, leading to $x=\ldots.$

3. Completing the square:

$x^2+bx+c=0$: $\left(x\pm\frac{b}{2}\right)^2\pm q\pm c=0$, $q\neq 0$ leading to $x=\ldots.$

Method marks for differentiation and integration:

1. Differentiation

Power of at least one term decreased by 1. $(x^n\to x^{n-1})$

2. Integration:

Power of at least one term increased by 1. $(x^n\to x^{n+1})$

Use of a formula:

Generally, the method mark is gained by **either**

quoting a correct formula and attempting to use it, even if there are mistakes in the substitution of values

or, where the formula is not quoted, the method mark can be gained by implication from the substitution of correct values and then proceeding to a solution.

Answers without working:

The rubric states "Without sufficient working, correct answers may be awarded no marks".

General policy is that if it could be done "in your head" detailed working would not be required. (Mark schemes may override this eg in a case of "prove or show...."

Exact answers:

When a question demands an exact answer, all the working must also be exact. Once a candidate loses exactness by resorting to decimals the exactness cannot be regained.

Rounding answers (where accuracy is specified in the question)
Penalise only once per question for failing to round as instructed - ie giving more digits in the answers. Answers with fewer digits are automatically incorrect, but the isw rule may allow the mark to be awarded before the final answer is given.

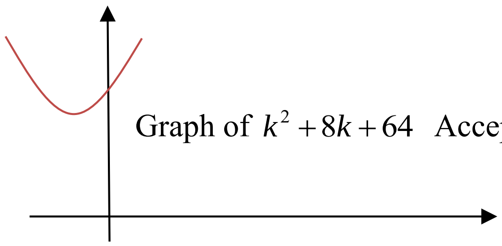
2306
4PM1 Paper 2R
Mark Scheme

Question	Scheme	Marks
1	$b^2 - 4ac = (k+8)^2 - 4\times 2\times k = [k^2 + 8k + 64]$	M1
	$k^2 + 8k + 64 = (k+4)^2 - 16 + 64 = (k+4)^2 + 48$	dM1A1
	Conclusion: Irrespective of the value of k, $(k+4)^2 \geqslant 0$ and so $b^2 - 4ac > 0$ therefore the equation will always have distinct (real) roots.	A1 [4]
		Total 4 marks

Mark	Notes
M1	Applies the discriminant on the given QE Ignore $=0,\ \ \geqslant 0,\ \ >0$ This must be correct for this mark $\left[(k+8)^2 - 4\times 2\times k\right]$. Ignore subsequent simplification errors.
dM1	Completes the square **ONLY** on **their** expression for the discriminant provided it is a 3TQ The expression must be of the form $(k+4)^2 \pm X$ where $X \neq 0, 64$ **NB:** This mark is dependent on the first M mark. Use of calculus to find $k = -4$ is M0
A1	For the correct expression.
A1	For a suitable conclusion. Only accept for this final mark a conclusion as follows [or an equivalent]. $(k+4)^2 \geqslant 0$ and so $(k+4)^2 + 48 > 0$ hence it will always have **(real), distinct** roots. Note: $(k+4)^2 + 48 \geqslant 0$ is A0

Alternative methods for last 3 marks.	
ALT 1 finds discriminant of $k^2+8k+64$	
M1	Finds discriminant of **their** $k^2+8k+64 \Rightarrow D = 64-4\times1\times64 = -192$
A1	Obtains the correct expression in terms of *k* with the correct value of *D*
A1	Conclusion: $D<0$ so $k^2+8k+64$ does not have real roots. k^2 is positive so we know it has a minimum so $k^2+8k+64>0$ This means that $k^2+8k+64>0$ and therefore the discriminant will be > 0 meaning that the equation has distinct real roots for all values of *k*.

ALT 2 – Uses calculus	
M1	$y=k^2+8k+64 \Rightarrow \frac{dy}{dk}=2k+8=0$ at max/min
A1	So coordinates of max/min are $(-4,\ 48)$ and finds second derivative $\frac{d^2y}{dk^2}=2$
A1	Conclusion: $\frac{d^2y}{dk^2}=2>0$ hence it is a minimum. $(-4,\ 48)$ is the minimum point and so $k^2+8k+64>0$ This means that $k^2+8k+64>0$ and therefore the discriminant will be > 0 meaning that the equation has distinct real roots for all values of *k*.

ALT 3 – Uses a graph	
M1	Draws a sketch.  Graph of $k^2+8k+64$ Accept anywhere in positive *y*
A1	The graph is placed correctly with the min point in the 4th quadrant.
A1	Conclusion: the graph is always positive with its min point as shown in the diagram. This means that $k^2+8k+64>0$ and therefore the discriminant will be > 0 meaning that the equation has distinct real roots for all values of *k*.

Question	Scheme	Marks
2(a)	$2(x+1)<5x-2 \Rightarrow 2x+2<5x-2 \Rightarrow 3x>4 \Rightarrow x>\frac{4}{3}$	M1A1 [2]
(b)	$3x^2-x-10=0 \Rightarrow (3x+5)(x-2)=0 \Rightarrow x=-\frac{5}{3}, 2$	M1
	$'-\frac{5}{3}' \leqslant x \leqslant '2'$ allow $'-\frac{5}{3}' < x < '2'$	dM1
	$-\frac{5}{3} \leqslant x \leqslant 2$	A1 [3]
(c)	$"\frac{4}{3}" < x\ '\leqslant'\ "2"$	B1ft [1]
		Total 6 marks

Part	Mark	Notes
(a)	M1	Attempts to solve the inequality with no more than one arithmetical error.
	A1	For the correct inequality.
(b)	M1	For attempting to solve the QE to find **two** critical values.
	For the dM and A marks allow any acceptable notation. For example; $-\frac{5}{3} \leqslant x \cap x \leqslant 2$ or, $-\frac{5}{3} \leqslant x$ **and** $x \leqslant 2$ The region must however indicate an INSIDE region	
	dM1	Forms an inside region with **their** cv's This mark is dependent on the first M mark in (b)
	A1	For the correct region with the correct inequalities.
(c)	B1ft	For the correct combined inside region. Penalise the incorrect inequality from part (b) $<$ in place of $\leqslant$ only once. Even if the **correct** inequality does not follow from their work, award this mark.

Question	Scheme	Marks
3(a)	$675 = \frac{\theta r^2}{2} \Rightarrow \theta = \frac{675 \times 2}{r^2} = \frac{1350}{r^2}$	M1
	$P = 2r + r\theta \Rightarrow P = 2r + r\left(\frac{1350}{r^2}\right) \Rightarrow P = 2r + \frac{1350}{r}$	M1A1 cso [3]
(b)	$\frac{dP}{dr} = 2 - \frac{1350}{r^2}$	M1
	$2 - \frac{1350}{r^2} = 0 \Rightarrow r = 15\sqrt{3}$ or $\sqrt{675}$	M1A1
	$P = 2 \times 15\sqrt{3} + \frac{1350}{15\sqrt{3}} = 60\sqrt{3}$	M1A1 [5]
(c)	$\frac{d^2P}{dr^2} = \frac{2700}{r^3}$ $r > 0 \Rightarrow \frac{d^2P}{dr^2} > 0 \Rightarrow$ minimum	M1A1ft [2]
		Total 10 marks

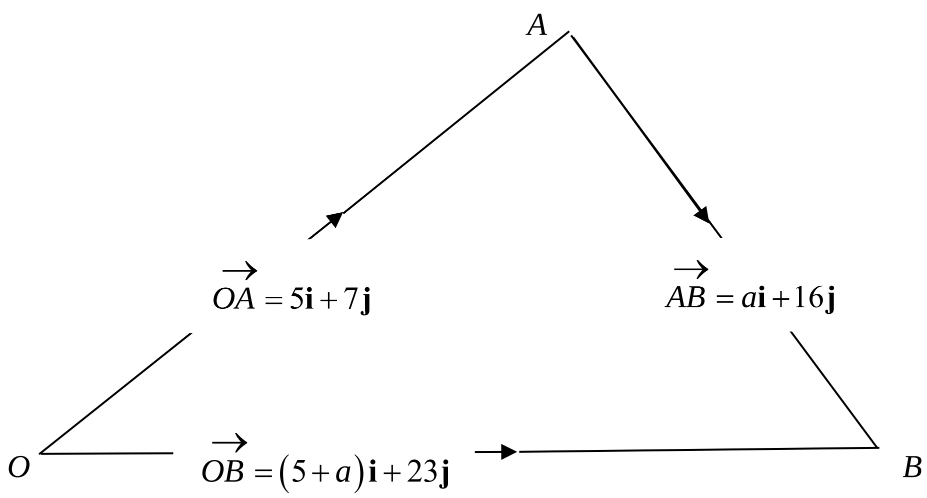
Part	Mark	Notes
(a)	**Note:** Accept any variable for *P* in this part of the question for the first two M marks **only** , even no variable as long as it is clear it is the perimeter.	
	M1	Applies the **correct** formula for the area of a sector with 675 cm^2 and attempts to rearrange to find an expression for θ, r or $r\theta$ Minimally acceptable expression: $\theta = \frac{k}{r^2}, r = \frac{k}{\theta r}$ or $\theta r = \frac{k}{r}$
	M1	Applies the **correct** formula for the perimeter of a sector and substitutes their expression for θ provided it is as a minimum of the form $\theta = \frac{k}{r^2}$ Minimally acceptable expression: $P = 2r + \frac{k}{r}$ where *k* is an integer
	A1	For a fully correct expression for *P* with no errors seen. You must see *P* = …….
	ALT – uses the formula $A = \frac{1}{2}rS$ where *S* is the arc length. *S* is very popular!	
	M1	Applies the **correct** formula for the area of a sector and rearranges to find an expression for *S*. $675 = \frac{rS}{2} \Rightarrow S = \frac{1350}{r}$ minimally acceptable $S = \frac{k}{r}$
	M1	Applies the correct formula for the perimeter of a sector. $P = 2r + S \Rightarrow P = 2r + \frac{1350}{r}$ minimally acceptable $P = 2r + \frac{k}{r}$
	A1	For a fully correct expression for *P* with no errors seen. You must see *P* = …….

	ALT 2 – works in degrees	
	M1	Applies a **correct** formula for the area of a sector in degrees and attempts to find an expression for θ $675=\frac{\theta}{360}\times\pi r^2 \Rightarrow \theta=\frac{675\times 360}{\pi r^2}=\frac{243000}{\pi r^2}$ Min: $\theta=\frac{K}{\pi r^2}$
	M1	Applies a **correct** formula for the perimeter of a sector and substitutes their expression for θ $P=2r+\frac{\theta}{360}\times 2\pi r\left(\frac{243000}{\pi r^2}\right)\Rightarrow P=2r+\frac{1350}{r}$ Min: $P=2r+\frac{K}{r}$ **Do not accept solution with a mix of degrees and radians.**
	A1	For a fully correct expression for P with no errors seen. You must see $P=$ …….
(b)	M1	**NB:** Allow poor notation here, even $\frac{dy}{dx}$ or nothing at all. For attempting to differentiate the **given** expression for P The minimally acceptable expression for the derivate is $\frac{dP}{dr}=2-\frac{Q}{r^2}$ where Q is a **positive** integer.
	M1	For setting their differentiated expression = 0 finding a value for r This is a simple equation to solve. Go through their working checking that it is correct. Do not award this mark for incorrect processing.
	A1	For $r=15\sqrt{3}$ oe [An approximate value is $r = 25.98$….]
	NB: Award the next 2 marks if they appear in part (b) only	
	M1	For substituting their value for r into the given expression for P Only allow this mark if: • They use the correct r and obtain the correct perimeter. • They use an incorrect r provided it is a positive value and **show explicit substitution**
	A1	For the correct final answer in exact form. There is no follow through here.
(c)	**NB: Award the next two marks if they appear in part (c) only.**	
	M1	Finds the second derivative. The minimally acceptable expression for the second derivative is $\frac{d^2P}{dr^2}=\frac{X}{r^3}$ where x is an integer. (The value for $\frac{d^2P}{dr^2}$ is awrt 0.15) If they test $\frac{dP}{dr}$ around the minimum point – send to Review. If they test P either side – score M0A0
	A1ft	For a correct conclusion. FT their 2nd derivative provided it is of the form $\frac{d^2P}{dr^2}=\frac{X}{r^3}$ r must be positive and if they find a value for $\frac{d^2P}{dr^2}$ then substitution **must** be seen unless they use $15\sqrt{3}$ and obtain awrt 0.15

Question	Scheme	Marks
4(a)	$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB} = 5\mathbf{i} + 7\mathbf{j} + a\mathbf{i} + 16\mathbf{j} = (5+a)\mathbf{i} + 23\mathbf{j}$	M1A1
	$\left(5\sqrt{29}\right)^2 = (5+a)^2 + 23^2 \Rightarrow 5 + a = \pm\sqrt{196} \Rightarrow a = 9, -19$	M1A1 [4]
(b)	$\overrightarrow{AB} = "9"\mathbf{i} + 16\mathbf{j} \Rightarrow \left\lvert\overrightarrow{AB}\right\rvert = \sqrt{"9"^2 + 16^2} = \sqrt{337}$	M1
	Unit vector: $\frac{1}{\sqrt{"337"}}("9"\mathbf{i} + 16\mathbf{j})$ oe.	A1 [2]
		Total 6 marks

Part	Mark	Notes
(a)	M1	For the correct vector statement for $\overrightarrow{OB}$ For example, accept $\overrightarrow{AB} = \overrightarrow{A0} + \overrightarrow{OB}$ This mark can be implied by a correct vector for $\overrightarrow{OB}$
	A1	For the correct vector in terms of a [simplified or unsimplified].
	M1	For using Pythagoras theorem with $\left(5\sqrt{29}\right)$ and their vector for $\overrightarrow{OB}$ and solving the equation to find **two** values of a
	A1	For the **two** correct values. $a = 9, -19$ seen in their working
(b)	M1	For finding |AB| by using a correct Pythagoras and writing down the unit vector where $\overrightarrow{AB} = k\mathbf{i} + 16\mathbf{j}$ where k is a **positive** value. Award for $\frac{"9"\mathbf{i} + 16\mathbf{j}}{\sqrt{"9"^2 + 16^2}}$ $\sqrt{9^2 + 16^2}$ can be implied by sight of $\sqrt{337}$ If they have an incorrect value for a, full substitution using Pythagoras theorem must be seen for the award of this mark.
	A1	For the correct unit vector. Accept any correct equivalent unit vectors. For example; $\frac{\sqrt{"337"}}{"337"}("9"\mathbf{i} + 16\mathbf{j})$ or $\frac{"9"\mathbf{i}}{\sqrt{"337"}} + \frac{16\mathbf{j}}{\sqrt{"337"}}$ Accept decimal answers. Eg., $0.054("9"\mathbf{i} + 16\mathbf{j})$ for awrt 0.054 Or $0.49\mathbf{i} + 0.87\mathbf{j}$ or better. Allow $\pm\frac{1}{\sqrt{"337"}}("9"\mathbf{i} + 16\mathbf{j})$

Useful Sketch



Question	Scheme	Marks
5(a)	$\frac{dv}{dt} = 4t - 19$	M1
	$\frac{dv}{dt} = 4 \times 5 - 19 = 1\left(\text{m/s}^2\right)$	A1 [2]
(b)	$2t^2 - 19t + 35 = 0 \Rightarrow (2t-5)(t-7) = 0 \Rightarrow t = \frac{5}{2}, 7$	M1
	$t_1 = \frac{5}{2} \quad t_2 = 7$	A1 [2]
(c)	$D = \int_{\frac{5}{2}}^{7}\left(2t^2 - 19t + 35\right)\,dt = \left[\frac{2t^3}{3} - \frac{19t^2}{2} + 35t\right]_{\frac{5}{2}}^{7}$	M1
	$\left(\frac{2\times 7^3}{3} - \frac{19\times 7^2}{2} + 35\times 7\right) - \left(\frac{2\times 2.5^3}{3} - \frac{19\times 2.5^2}{2} + 35\times 2.5\right)$	M1
	$= \left[\frac{49}{6} - \frac{925}{24}\right] = [8.167 - 38.542] = \left[-\frac{243}{8} \text{ or -30.375}\right]$	A1 [3]
	$\Rightarrow D = \frac{243}{8}$ (m) oe e.g. 30.375	
		Total 7 marks

Part	Mark	Notes
(a)	M1	For differentiating the given *v* and substituting in $t = 5$ into the derivative. Both differentiation and substitution must be correct for this mark.
	A1	For the correct acceleration of 1 m/s^2. Units are not required.
(b)	M1	For attempting to solve the given 3TQ for the velocity. A correct method must be used, and they must reach **two** values of *t* for the award of this mark. See General Guidance for the definition of an attempt to solve a 3TQ. If there is no visible method seen, both values of *t* must be seen for the award of this mark.
	A1	For the two correct values of *t* They do not need to ne identified as t_1 or t_2 for this mark. Accept $\frac{5}{2}$ and 7 seen.
(c)	M1	For an attempt to integrate the given expression for *v*. At least one term must be correct and no terms are to be differentiated. If the value of 35 'disappears' it is M0. The question states 'use calculus' so integration must be seen.
	M1	For substituting the **two** values of *t* into their integrated expression. and subtracting the result (either way) of both substitutions. Allow for any changed expression from *v* A correct answer of $\pm\frac{243}{8}$ or ± 30.375 implies correct substitution into a correct integral. If the integration is incorrect, or the values are incorrect, full correct substitution must be seen for the award of this mark.
	A1	For the correct distance (which must be a positive value).

Question	Scheme	Marks
6(a)	$\alpha+\beta=-\frac{5}{2} \quad \alpha\beta=-\frac{p}{2}$	B1
	$\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$	M1
	$\alpha^3+\beta^3=\left(-\frac{5}{2}\right)^3-3\left(-\frac{p}{2}\right)\left(-\frac{5}{2}\right)=-\frac{125}{8}-\frac{15p}{4}=-\frac{215}{8}$	M1M1
	$\Rightarrow p=3$	A1
	ALT	[5]
	$\alpha^3+\beta^3=(\alpha+\beta)(\alpha^2-\alpha\beta+\beta^2)=(\alpha+\beta)\left((\alpha+\beta)^2-3\alpha\beta\right)$	[M1]
(b)	<u>Sum:</u>	
	$\frac{\alpha+\beta}{\alpha^2}+\frac{\alpha+\beta}{\beta^2}=\frac{\alpha\beta^2+\beta^3+\alpha^3+\alpha^2\beta}{\alpha^2\beta^2}=\frac{\alpha^3+\beta^3+\alpha\beta(\alpha+\beta)}{\alpha^2\beta^2}$	M1
	ALT	
	$\frac{\alpha+\beta}{\alpha^2}+\frac{\alpha+\beta}{\beta^2}=\frac{\alpha^2(\alpha+\beta)+\beta^2(\alpha+\beta)}{\alpha^2\beta^2}=\frac{(\alpha^2+\beta^2)(\alpha+\beta)}{\alpha^2\beta^2}$	M1
	$=\frac{\left((\alpha+\beta)^2-2\alpha\beta\right)(\alpha+\beta)}{\alpha^2\beta^2}$	
	$\frac{-\frac{215}{8}+\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{\left(-\frac{3}{2}\right)^2}=-\frac{185}{18}$ allow $\frac{185}{18}$ if they start with a negative.	B1
	<u>Product:</u>	
	$\left(\frac{\alpha+\beta}{\alpha^2}\right)\times\left(\frac{\alpha+\beta}{\beta^2}\right)=\frac{(\alpha+\beta)^2}{\alpha^2\beta^2}=\frac{\left(-\frac{5}{2}\right)^2}{\left(-\frac{3}{2}\right)^2}=\frac{25}{9}$	M1A1 [5]
	<u>Equation:</u>	
	$x^2+\frac{185}{18}x+\frac{25}{9}=0\Rightarrow 18x^2+185x+50=0$ oe	
	Total 10 marks	

Part	Mark	Notes
(a)	B1	For the correct expression/values for **BOTH** the sum and product. This must be identified, or implied from their working.
	M1	For the correct algebra to find $\alpha^3+\beta^3$ in terms of $\alpha+\beta$ and $\alpha\beta$ • $\alpha^3+\beta^3=(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)$ • $\alpha^3+\beta^3=(\alpha+\beta)\left((\alpha+\beta)^2-3\alpha\beta\right)$ • Or any other algebra, but do not award this mark until the values of $\alpha+\beta$ and $\alpha\beta$ can be substituted in directly.
	M1	For substituting their values of the sum and product into their expression for $\alpha^3+\beta^3$
	M1	For solving the **linear** equation in *p* Allow one slip in their working.
	A1	For the correct value of *p* with no errors.
(b)	M1	For the correct algebra for the sum of roots. This must be such that the given value of $\alpha^3+\beta^3$, with their values of $\alpha\beta$ and $(\alpha+\beta)$ can be substituted in. If they use the **ALT** they will not need $\alpha^3+\beta^3$ Some candidates will reverse the sign at this stage in anticipation of the reversal required in the equation.
	M1	For substituting in the given value for $\alpha^3+\beta^3$, and their values for $\alpha+\beta$ and $\alpha\beta$ into their expansion for the sum. If they use the **ALT** they will not need $\alpha^3+\beta^3$
	B1ft	For the correct value of the product of roots. You must follow through their *p*. You do not need to check the calculation once you see the correct algebra with a correct substitution.
	M1	For forming an equation with their sum and product. Ft the sign of their sum. Some candidates reverse the sign when finding the sum. Watch out for that! Accept this without = 0
	A1	For a correct equation including = 0 There is no follow through on this mark. **NB** accept a correct equivalent equation provided it has integer coefficients. e.g. $36x^2+370x+100=0$
	SC – they solve the equation and find values for α and β Award marks for correct work seen above in part (b) • If they do not obtain an expansion for the sum into which $\alpha^3+\beta^3$, $\alpha\beta$ and $(\alpha+\beta)$ cannot be directly substituted – first M0 • If they do not substitute $\alpha^3+\beta^3$, $\alpha\beta$ and $(\alpha+\beta)$ but other values based on α and β - second M0 • If they cannot substitute $\alpha\beta$ and $(\alpha+\beta)$ into the product – B0 Then allow marks for forming the equation as above.	

Question	Scheme	Marks
7(a)	$\left(\cos 3\theta+\sqrt{3}\sin 3\theta\right)^{\frac{1}{2}}=0 \Rightarrow \cos 3\theta=-\sqrt{3}\sin 3\theta \Rightarrow \tan 3\theta=-\frac{1}{\sqrt{3}}$	M1
	$\Rightarrow 3\theta=-\frac{\pi}{6}$ **or** $\frac{5\pi}{6}$	A1
	$\Rightarrow m=-\frac{\pi}{18} \quad n=\frac{5\pi}{18}$	A1 [3]
(b)	$V=\pi\int_{-\frac{\pi}{18}}^{\frac{5\pi}{18}}\left(\cos 3\theta+\sqrt{3}\sin 3\theta\right)\,\mathrm{d}\theta$	M1
	$V=\pi\left[\frac{\sin 3\theta}{3}-\frac{\sqrt{3}\cos 3\theta}{3}\right]_{-\frac{\pi}{18}}^{\frac{5\pi}{18}}$	M1
	$V=\pi\left[\left(\frac{\sin 3\left(\frac{5\pi}{18}\right)}{3}-\frac{\sqrt{3}\cos 3\left(\frac{5\pi}{18}\right)}{3}\right)-\left(\frac{\sin 3\left(-\frac{\pi}{18}\right)}{3}-\frac{\sqrt{3}\cos 3\left(-\frac{\pi}{18}\right)}{3}\right)\right]$	M1
	$V=\frac{4\pi}{3}$	A1 [4]
		Total 7 marks

Part	Mark	Notes
(a)	M1	Sets the equation = 0 and obtains $\tan 3\theta=k$
	A1	Finds at least one correct value of 3θ $3\theta=-\frac{\pi}{6}$ **or** $\frac{5\pi}{6}$ Works in degrees Accept -30° **or** 150° for this mark. This mark is also implied by one correct solution for *m* or *n* in degrees or radians. **NB This is an M mark in Epen**
	A1	For $m=-\frac{\pi}{18} \quad n=\frac{5\pi}{18}$ which must be in radians. Accept embedded in coordinates. *M* and *n* do not need to be identified.

(b)	**Working in degrees**. Allow working in degrees up to the last M mark.	
	M1	For a correct statement for the volume of revolution with π and their limits. Allow $V = \pi\int_{'-\frac{\pi}{18}'}^{'\frac{5\pi}{18}'}\left(\left[\cos 3\theta + \sqrt{3}\sin 3\theta\right]^{\frac{1}{2}}\right)^2 \mathrm{d}\theta$ or $V = \pi\int_{'-\frac{\pi}{18}'}^{'\frac{5\pi}{18}'}\left(\cos 3\theta + \sqrt{3}\sin 3\theta\right) \mathrm{d}\theta$ Allow also: $V = \pi\int_{'-\frac{\pi}{18}'}^{'\frac{5\pi}{18}'}\left(\cos 3\theta + \sqrt{3}\sin 3\theta\right) \mathrm{d}x$ or even $V = \pi\int_{'-\frac{\pi}{18}'}^{'\frac{5\pi}{18}'}\left(\cos 3\theta + \sqrt{3}\sin 3\theta\right)$ **This mark can be implied by correct further working** <u>Working in degrees</u> $V = \pi\int_{'-10^\circ'}^{'50^\circ'}\left(\cos 3\theta + \sqrt{3}\sin 3\theta\right) \mathrm{d}\theta$
	M1	For an acceptable attempt at integration. Minimally acceptable integration is as follows. $\cos 3\theta \Rightarrow \pm\frac{\sin 3\theta}{3}, \quad \sin 3\theta \Rightarrow \pm\frac{\cos 3\theta}{3}$ Ignore absence or incorrect limits and the absence of π for this mark.
	M1	For substitution of the correct limits into their integrated expression the correct way around. This must be a changed expression from the one given. If the integrated expression is correct with correct limits, allow a final volume of $\frac{4\pi}{3}$ seen without explicit substitution. If the final volume is incorrect without evidence of explicit substitution award M0. If the integrated expression is incorrect or the limits are incorrect, explicit substitution **must** be seen for the award of this mark. **Do not** allow use of degrees at this stage. Ignore absence of π for this mark.
	A1	For the correct volume $(V) = \frac{4\pi}{3}$

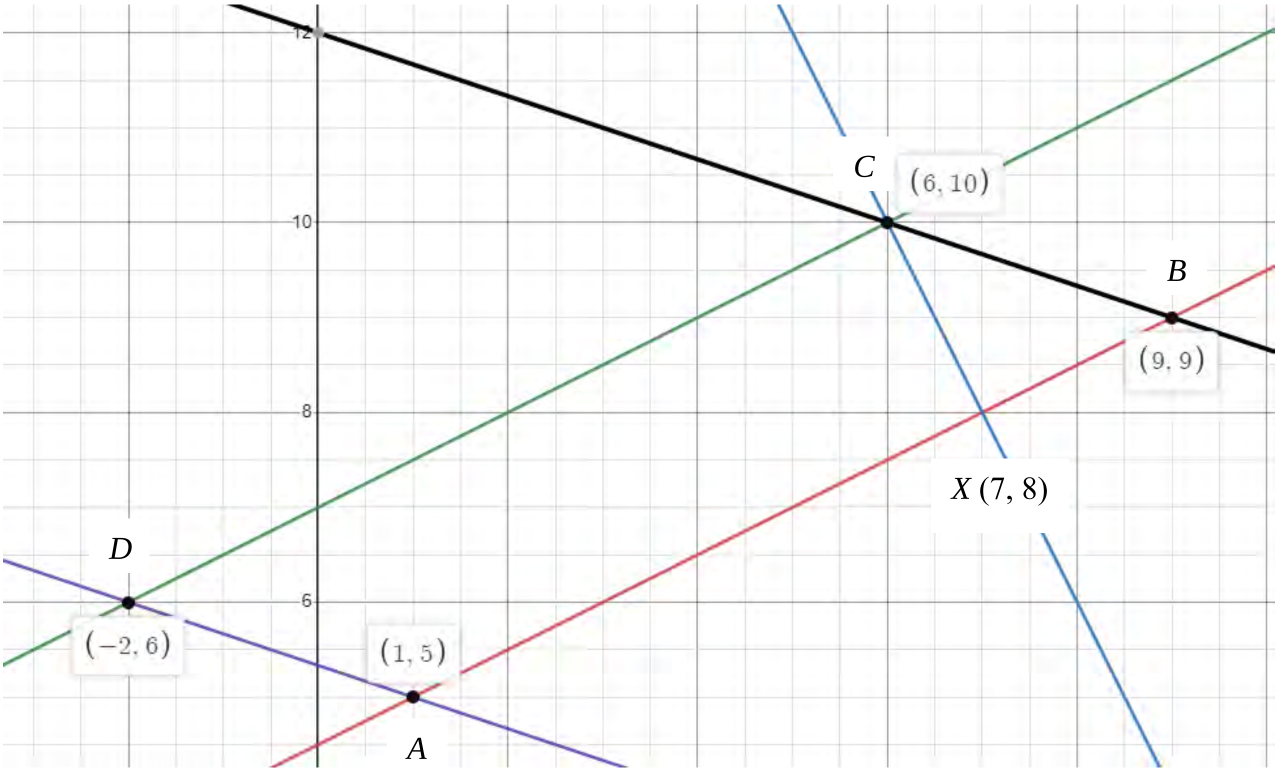
Question	Scheme	Marks
8(a)	$\dfrac{y-5}{5-9}=\dfrac{x-1}{1-9}$ $\Rightarrow x-2y+9=0$	M1A1 A1 [3]
(b)	Coordinates of point X $\left(\dfrac{3\times 9+1\times 1}{3+1},\dfrac{3\times 9+1\times 5}{3+1}\right)=(7,8)$ The perpendicular gradient = $-\dfrac{1}{\frac{1}{2}}=-2$ Equation of l $y-8=-2(x-7)\Rightarrow y=-2x+22$ *	B1B1 B1ft M1A1 cso [5]
(c)	$y=-2(6)+22\Rightarrow p=10$	B1 [1]
(d)	$\overrightarrow{BA}=\begin{pmatrix}1-9\\5-9\end{pmatrix}=\begin{pmatrix}-8\\-4\end{pmatrix}\Rightarrow \overrightarrow{CD}=\begin{pmatrix}6-8\\10-4\end{pmatrix}=\begin{pmatrix}-2\\6\end{pmatrix}$ and coordinates of C are (6, 10) so coordinates of D are (– 2, 6)	M1 A1A1 [3]
(e)	Length of AB (or CD) = $\sqrt{(9-5)^2+(9-1)^2}=\sqrt{80}$ Length of CX = $\sqrt{(10-8)^2+(7-6)^2}=\sqrt{5}$ Area of parallelogram $ABCD$ = $\sqrt{5}\times\sqrt{80}=\sqrt{400}=20$ (units^2)	B1 B1 M1A1 [4]
		Total 16 marks

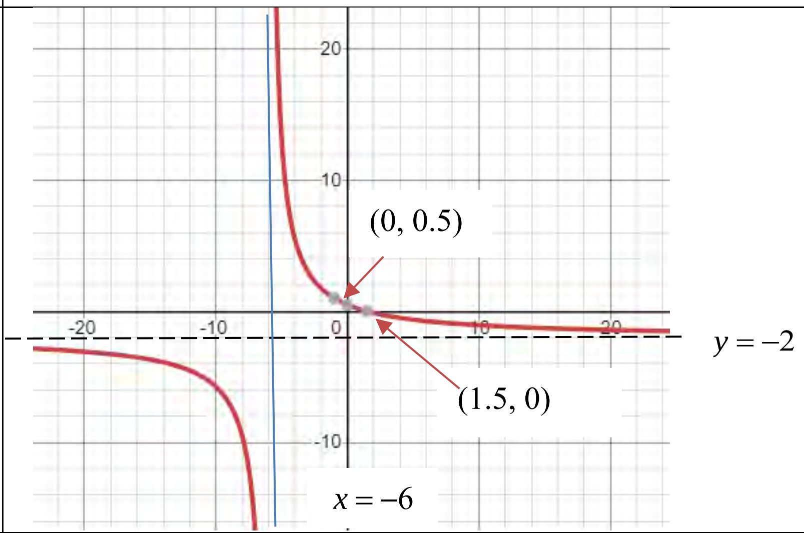
Part	Mark	Notes
(a)	M1	For using a correct method and the given coordinates of A and B to form the equation of AB. Do not score this mark until they find the gradient and form the equation of the line using a correct formula. If they use $y=mx+c$ do not allow this mark until they find c and form a complete equation. $\left[m=\dfrac{1}{2},\quad c=\dfrac{9}{2}\quad y=\dfrac{x}{2}+\dfrac{9}{2}\right]$
	A1	For the correct equation of AB in any form.
	A1	For the correct equation of AB in the required form.

(b)	B1	For either x or y correct coordinates of point X **NB This is an M mark in Epen**
	B1	For both x and y correct coordinates of point X **NB This is an A mark in Epen.**
	B1ft	For writing down the inverse reciprocal of their gradient for AB Ft their gradient form part (a)
	M1	For forming an equation for l using their coordinates of X and their negative reciprocal gradient of AB $y-8=-2(x-7)\Rightarrow y=-2x+22$
	A1	For the correct equation of l as shown.
(c)	B1	For $y = 10$
(d)	M1	For a suitable method. **Method 1 - Uses vectors:** $\overrightarrow{BA}=\begin{pmatrix}1-9\\5-9\end{pmatrix}=\begin{pmatrix}-8\\-4\end{pmatrix}\Rightarrow \overrightarrow{CD}=\begin{pmatrix}6+(-8)\\10+(-4)\end{pmatrix}\Rightarrow\left[\begin{pmatrix}-2\\6\end{pmatrix}\right]$ OR: $\overrightarrow{BC}=\begin{pmatrix}6-9\\9-10\end{pmatrix}=\begin{pmatrix}-3\\-1\end{pmatrix}\Rightarrow \overrightarrow{AD}=\begin{pmatrix}1-3\\5-(-1)\end{pmatrix}\Rightarrow\left[\begin{pmatrix}-2\\6\end{pmatrix}\right]$ **Method 2 - Uses simultaneous equations**: The equation of AD is $y=-\frac{x}{3}+\frac{16}{3}$ and of CD is $y=\frac{x}{2}+7$ $-\frac{x}{3}+\frac{16}{3}=\frac{x}{2}+7\Rightarrow x=-2$ and $y=6$ **Method 3 - Uses the gradient and length of *AB* or *CD*** $AB=CD=4\sqrt{5}=\sqrt{(6-x)^2+(y-10)^2}$ Gradient: $\frac{1}{2}=\frac{10-y}{6-x}\Rightarrow x=2y-14$ $(4\sqrt{5})^2=(6-(2y-14))x^2+(y-10)^2\Rightarrow 5y^2-100y+420=0$ $\Rightarrow y=6,14$ and $x=-2,14$ Allow no more than one error in either method.
	A1	For either correct x or y coordinate of D [(– 2, 6)] **NB** this is an A mark in Epen
	A1	For both correct coordinates of D (– 2, 6)

(e)	B1	For the correct length of either ***AB* [*CD*] or *CX*** These are given coordinates there is no ft
	B1	For the correct lengths of both ***AB* [*CD*] and *CX*** Ft their C and their X
	M1	For using a correct method to calculate the area of a parallelogram. $\text{b} \times \text{h} = CX \times AB = \sqrt{5} \times \sqrt{80} = \sqrt{400} = 20$ If their lengths are incorrect allow this mark provided their lengths are identified as base and perpendicular height. If they use AD as the base $\left[\sqrt{10}\right]$, the perpendicular height required is $2\sqrt{10}$ so base × height = $\sqrt{10} \times 2\sqrt{10} = ...$
	A1	For the correct area of 20 [units^2]
	ALT – Using determinants	
	B1B1	For sight of the correct array which must use the coordinates of A, B, C and D **ONLY.** The coordinates (7, 8) seen in the array is B0B0 $\begin{pmatrix} 1 & 9 & '6' & '-2' & 1 \\ 5 & 9 & '10' & '6' & 5 \end{pmatrix}$ Award both marks for fully correct. Award B1 if there are no more than 2 errors but with none missing. There must be 5 sets of coordinates in the array with first and last the same. **The coordinates must go in order around the parallelogram clockwise or anticlockwise.**
	M1	For the correct evaluation of their 2×5 array. Allow this even if they have (7, 8) included instead of (9, 9) [which is a common error] $\frac{1}{2}\begin{pmatrix} 1 & 9 & 6 & -2 & 1 \\ 5 & 9 & 10 & 6 & 5 \end{pmatrix} = \frac{1}{2}\left\|\left[(9+90+36-10)-(45+54-20+6)\right]\right\| = ...$
	A1	For the correct area of 20 [units^2]

Useful sketch



Question	Scheme	Marks
9(a)	(i) $y=-2$ (ii) $x=-6$	B1 B1 [2]
(b)	(i) $\left(\frac{3}{2}, 0\right)$ (ii) $\left(0, \frac{1}{2}\right)$	B1 B1 [2]
(c)		B1 – shape B1ft– Asymptotes B1ft – Intersections [3]
(d)	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{(x+6)(-2)-(3-2x)(1)}{(x+6)^2}$ $\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{-15}{(x+6)^2}$ with conclusion; numerator negative, denominator always positive, $\frac{-}{+}\Rightarrow$ negative	M1A1 A1 [3]
(e)	$\frac{-15}{(x+6)^2}=-\frac{3}{5}\Rightarrow 25=(x+6)^2\Rightarrow x=-6\pm 5=-11,\quad -1$ $y=\frac{3-2(-1)}{-1+6}=1$ $y-1=-\frac{3}{5}(x-[-1])\Rightarrow y=-\frac{3}{5}x+\frac{2}{5}\Rightarrow k=\frac{2}{5}$	M1A1 B1 M1A1 [5]
		Total 15 marks

Part	Mark	Notes
(a) (i)	B1	For the correct equation. If their equations are not labelled (i) and (ii) accept them in the order given only. For example, the following presentation is B0B0 $x = -6$ $y = -2$
(ii)	B1	For the correct equation.
(b)		If their coordinates are not labelled (i) and (ii) or are given in the incorrect place [for example, (b)(i) $y = \frac{1}{2}$ is B0] accept them in the order given only. For example, the following presentation is B0B0 $\left(0, \frac{1}{2}\right)$ $\left(\frac{3}{2}, 0\right)$
(i)	B1	For the correct coordinates. Accept $x = \frac{3}{2}$
(ii)	B1	For the correct coordinates. Accept $y = \frac{1}{2}$
(c)	B1	For the correct shape with two branches the correct way around anywhere in the grid. It must be asymptotic in nature. The ends of the curve must not come back on themselves. Whilst you need to be fairly generous, any obvious turning back is B0
	B1ft	For the correct asymptotes drawn and labelled with at least one branch of the curve [which must be asymptotic in nature in at least one branch] in the correct place seen. Ft their asymptotes Accept the vertical line drawn shown passing through their -6 and the horizontal line drawn shown passing through their -2
	B1ft	For the correct coordinates of intersections with the relevant branch of the curve drawn correctly seen. The curve must go through the points. Do not accept touching the axis. Allow $\frac{1}{2}$ marked on *y*-axis and $\frac{3}{2}$ marked on the *x*-axis. Ft their coordinates of intersections.
(d)	M1	For an attempt at quotient rule. • The denominator must be squared, or accept $(x+6)(x+6)$ or $x^2+12x+36$ • Both $(x+6)$ and $(3-2x)$ differentiated correctly. • The two terms in the numerator subtracted either way around.
	A1	Fully correct derivative (simplification not required for this mark).
	A1	For a correct simplified derivative with a correct conclusion. For example: $(x+6)^2 \geqslant 0$, -15 is negative, $\frac{\text{negative}}{\text{positive}}$ is always negative. [Accept also $(x+6)^2 > 0$]

(e)	M1	For setting the value of $-\frac{3}{5}=$ their $\frac{\mathrm{d}y}{\mathrm{d}x}$ with an attempt to find at least one value of x. Allow this even if their derivative results in a linear equation.
	A1	For **both** correct values of x
	B1	For $y=1$ using $x=-1$ OR For $y=-5$ using $x=-11$
	M1	For forming an equation of the line with either $x=-1,\quad y=1$ or $x=-11,\quad y=-5$ or their x and their y
	A1	For the correct value of k (accept an embedded value). $y=-\frac{3}{5}x+\frac{2}{5}\Rightarrow k=\frac{2}{5}$ You can award this mark even if the previous A mark has not been scored. So, for a correct solution without showing that $x=-11$ score M1A0B1M1A1
	ALT for last 3 marks	
	B1	Sets $-\frac{3}{5}x+k=\frac{3-2x}{x+6}$
	M1	Substitutes $x=-1$ or -11 into the above equation
	A1	For $k=\frac{2}{5}$ You can award this mark even if the previous A mark has not been scored. So, for a correct solution without showing that $x=-11$ score M1A0B1M1A1

Question	Scheme	Marks
10	$8\log_x 64 = \frac{8\log_4 64}{\log_4 x}$	M1
	$\log_4 x^3 = 3\log_4 x$	M1
	$\log_4 x^3 + 8\log_x 64 = 22 \Rightarrow 3\log_4 x + \frac{8\log_4 64}{\log_4 x} = 22$	
	$\Rightarrow 3(\log_4 x)^2 + 8\log_4 64 = 22\log_4 x \Rightarrow 3(\log_4 x)^2 - 22\log_4 x + 24 = 0$	M1
	$3(\log_4 x)^2 - 22\log_4 x + 24 = 0 \Rightarrow (3\log_4 x - 4)(\log_4 x - 6) = 0$	M1
	$\Rightarrow \log_4 x = \frac{4}{3},\quad 6$	A1
	$x = 4^{\frac{4}{3}}$ or awrt 6.35 and $x = 4096$	M1A1 [7]
		Total 7 marks

NOTE WELL! This can be solved using a modern calculator. No working = no marks. Award marks only for work explicitly seen.

Mark	Notes
Works in base 4	
M1	For changing the base of the log correctly to base 4 $3\log_4 x + 8\log_x 64 = 22 \Rightarrow 3\log_4 x + \frac{8\log_4 4^3}{\log_4 x} = 22$
M1	For applying the power law correctly seen anywhere in their work. This mark can also be awarded for explicit application of the power law on $\log_4 64 = 3\log_4 4$
M1	For multiplying through by $\log_4 x$ and forming a 3TQ in log base 4
M1	For solving their 3TQ by any valid and correct method. If there is no method seen with an incorrect 3TQ or with incorrect solutions following a correct 3TQ this is M0 They must obtain two values for their log.
A1	For both correct values of log x $\left[\frac{4}{3},\quad 6\right]$
M1	For undoing either log correctly. Allow this mark for any erroneous log they find, but undo correctly.
A1	For **both** values of x Accept $4^{\frac{4}{3}}$ or awrt 6.35 and 4^6 or 4096

Works in base *x*	
M1	For changing the base of the log correctly to base *x* or vice versa. $\log_4 x^3 + 8\log_x 64 = 22 \Rightarrow \dfrac{3\log_x x}{\log_x 4} + 8\log_x 4^3 = 22$
M1	For applying the power law correctly seen anywhere in their work. This mark can also be awarded for explicit application of the power law on $\log_x 64 = 3\log_x 4$
M1	For multiplying through by $\log_x 4$ and forming a 3TQ in log base *x* $\dfrac{3}{\log_x 4} + 24\log_x 4 = 22 \Rightarrow 24(\log_x 4)^2 - 22\log_x 4 + 3 = 0$
M1	For solving their 3TQ by any valid and correct method. If there is no method seen with an incorrect 3TQ or with incorrect solutions following a correct 3TQ this is M0 $24(\log_x 4)^2 - 22\log_x 4 + 3 = 0 \Rightarrow (4\log_x 4 - 3)(6\log_x 4 - 1) = 0 \Rightarrow \log_x 4 = ..., ...$ They must obtain two values for their log.
A1	For both correct values. $\log_x 4 = \dfrac{3}{4}, \dfrac{1}{6}$
M1	For undoing either log correctly, but they **must** obtain a value for *x* = ….. $x = 4^{\frac{4}{3}}$ or $x = 4^6$ Allow this mark for any erroneous log they find, but undo correctly.
A1	Accept $4^{\frac{4}{3}}$ or awrt 6.35 and 4^6 or 4096
Works in base 2	
M1	Changes the base of **at least one** log to base 2. $\dfrac{3}{2}\log_2 x$, $\dfrac{8\log_x x^6}{\log_2 x}$
M1	For applying the power law correctly seen anywhere in their work. This mark can also be awarded for explicit application of the power law on $\log_2 64 = 6\log_2 2$
M1	For multiplying through by $\log_2 x$ and forming a 3TQ in base 2 $\dfrac{3}{2}(\log_2 x)^2 - 22\log_2 x + 48 = 0 \Rightarrow \left[3(\log_2 x)^2 - 44\log_2 x + 96 = 0\right]$
M1	For solving their 3TQ by any valid and correct method. If there is no method seen with an incorrect 3TQ or with incorrect solutions following a correct 3TQ this is M0 $(12\log_2 x - 1)(8\log_2 x - 3) = 0 \Rightarrow \log_2 x = ..., ...$
A1	For both correct values. $\log_2 x = \dfrac{8}{3}, 12$
M1	For undoing either log correctly. $x = 2^{\frac{8}{3}}$ or $x = 2^{12}$ Allow this mark for any erroneous log they find, but undo correctly.
A1	Accept $2^{\frac{8}{3}}$ or awrt 6.35 and 2^{12} or 4096

Question	Scheme	Marks
11(a)	$\cos 2A = \cos^2 A - \sin^2 A \Rightarrow \cos 2A = \left(1-\sin^2 A\right) - \sin^2 A = 1 - 2\sin^2 A$	M1M1
	$2\sin^2 A = 1 - \cos 2A \Rightarrow \sin^2 A = \frac{1}{2}\left(1-\cos 2A\right)$ *	A1cso [3]
(b)	$\cos^2 A = \frac{1}{2}\left(1+\cos 2A\right)$	B1
	$\sin^4 x + \cos^4 x = \left(\left(\frac{1-\cos 2x}{2}\right)^2 + \left(\frac{1+\cos 2x}{2}\right)^2\right)$	
	$= \frac{1}{4}\left(\left(1-\cos 2x\right)^2 + \left(1+\cos 2x\right)^2\right)$	
	$= \frac{1}{4}\left(1 - 2\cos 2x + \cos^2 2x + 1 + 2\cos 2x + \cos^2 2x\right)$	M1
	$= \frac{1}{4}\left(2 + 2\cos^2 2x\right) = \left\{\frac{1}{2}\left(1+\cos^2 2x\right)\right\}$	A1
	$\left\{\cos^2 2x = \frac{1+\cos 4x}{2}\right\}$	
	$\sin^4 x + \cos^4 x = \frac{1}{2}\left(1 + \frac{1+\cos 4x}{2}\right) = \frac{3+\cos 4x}{4}$ *	M1A1 cso [5]
(c)	$5\sin 2\theta + 6 = 8\left(\frac{3+\cos 2\theta}{4}\right) \Rightarrow 5\sin 2\theta + 6 = 6 + 2\cos 2\theta$	M1
	$\Rightarrow 5\sin 2\theta = 2\cos 2\theta \Rightarrow \frac{\sin 2\theta}{\cos 2\theta} = \frac{2}{5} \Rightarrow \tan 2\theta = \frac{2}{5}$	M1
	$2\theta = 21.801...^\circ, 201.801...^\circ, 381.801...^\circ$	A1
	$\Rightarrow \theta = 10.9^\circ, 100.9^\circ$ Penalise extra angles in range by withholding the final A mark. Extra angles out of range – ignore.	A1 [4]
		Total 12 marks

Part	Mark	Notes
(a)	M1	For using the summation formula for cos 2*A* They must start with either $\cos 2A = \cos^2 A - \sin^2 A$ or $\cos\left(A+A\right) = \cos A\cos A - \sin A\sin A$
	M1	For eliminating $\cos^2 A$ using $\cos^2 A + \sin^2 A = 1$ and attempting to rearrange to the required result. This is not dependent on the first M mark, so if they start with another identity for cos2*A* they can still get this mark.
	A1 cso	For the correct identity with no errors seen. This is a given result.
	NB Some candidates work backwards – that is fine, please follow their working.	

Working with a different variable. If they work in this part with a different variable (eg A) then award all the marks as appropriate up to the last mark. If they leave their final answer in terms of another variable, withhold the final A mark only. If they however, change to x on the final line award all the marks [provided everything is correct]		
(b)	**Main method**	
	B1	For use of the correct identity for $\cos^2 x$ $\left[\cos^2 x = \frac{1}{2}(1+\cos 2x)\right]$ or if they convert $\cos^2 x$ to $\sin^2 x$ use the Pythagorean identity $\cos^2 x = 1-\sin^2 x$ and apply the given identity.
	M1	For squaring both identities. This must be a correct expansion. Ft their identity for $\left[\cos^2 x = \frac{1}{2}(1+\cos 2x)\right]$
	A1	For collecting like terms and obtaining $\frac{1}{2}(1+\cos^2 2x)$ oe. For example, $\frac{1}{4}(2+2\cos^2 2x)$
	M1	For applying the identity: $\cos^2 A = \frac{1}{2}(1+\cos 2A) \Rightarrow \left[\cos^2 2A = \frac{1}{2}(1+\cos 4A)\right]$ on $\cos^2 2x$ **only** again to achieve an expression in cos 4x **only**
	A1 cso	For the correct identity with no errors seen. This is a given result. You must check every line of their work carefully.
	ALT 1	
	B1	For use of the correct identity for $\sin 2A = 2\sin A\cos A$ [seen later in their working].
	M1	For using the expansion of $\sin^2 x + \cos^2 x$ as follows $\sin^4 x + \cos^4 x = (\sin^2 x + \cos^2 x)^2 - 2\sin^2 x\cos^2 x$ This must be correct
	A1	For obtaining $\sin^4 x + \cos^4 x = 1 - \frac{1}{2}\sin^2 2x$ oe. For example; $\frac{1}{2}(2-\sin^2 2x)$
	M1	For applying the given identity on $\sin^2 2x$ **only** to achieve an expression in cos 4x **only** $\sin^2 A = \frac{1}{2}(1-\cos 2A) \Rightarrow \left[\sin^2 2A = \frac{1}{2}(1-\cos 4A)\right]$ $\sin^4 x + \cos^4 x = 1 - \frac{1}{2}\left(\frac{1-\cos 4x}{2}\right) \Rightarrow \left(\sin^4 x + \cos^4 x = \frac{3+\cos 4x}{4}\right)$
	A1 cso	For the correct identity with no errors seen. This is a given result. You must check every line of their work carefully.

	ALT 2 – Works backwards from the given result	
	B1	For use of the correct identity for $\sin 2A = 2\sin A\cos A$ [seen later in their working].
	M1	Applies the cos2*A* identity and converts 3 into $3\left(\sin^2 2x + \cos^2 2x\right)$ $\frac{3+\cos 4x}{4} = \frac{3\sin^2 2x + 3\cos^2 2x + \left(\cos^2 2x - \sin^2 2x\right)}{4}$
	A1	Obtains $\frac{3+\cos 4x}{4} = \frac{4\cos^2 2x + 2\sin^2 2x}{4}$
	M1	Applies the cos2*A* identity and expands the bracket. $\frac{3+\cos 4x}{4} = \frac{4\left(\cos^2 x - \sin^2 x\right)^2 + 8\sin^2 x\cos^2 x}{4}$ $= \frac{4\cos^4 x - 8\sin^2 x\cos^2 x + 4\sin^4 x + 8\sin^2 x\cos^2 x}{4}$
	A1 cso	Simplifies to the required result with no errors seen $\frac{3+\cos 4x}{4} = \sin^4 x + \cos^4 x$ *

(c)	M1	For obtaining the correct equation in terms of $\sin 2\theta$ and $\cos 2\theta$ $6 + 2\cos 2\theta = 5\sin 2\theta + 6$ Accept unsimplified, accept even: $8\left(\frac{3+\cos 2\theta}{4}\right) = 5\sin 2\theta + 6$
	M1	For using the $\tan 2\theta = \frac{\sin 2\theta}{\cos 2\theta}$ identity correctly on their expression following an expression in the form $A\cos 2\theta = B\sin 2\theta$ and must be in terms of 2θ
	A1	For achieving at least one correct angle for 2θ **NB This is an M mark in Epen**
	A1	For awrt both 10.9° and 100.9° Penalise extra angles in range by withholding the final A mark. Extra angles out of range – ignore.

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